

Nariai–Bertotti–Robinson spacetimes as a building material for one-way wormholes with horizons, but without singularity

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Abstract

We discuss the problem of wormholes from the viewpoint of gluing together two Reissner–Nordström-type universes while putting between them a segment of the Nariai-type world (in both cases there are also present electromagnetic fields as well as the cosmological constant). Such a toy wormhole represents an example of one-way topological communication free from causal paradoxes, though involving a travel to next spacetime sheet since one has to cross at least a pair of horizons through which the spacetimes’ junction occurs. We also consider the use of thin shells in these constructions. Such a “material” for wormholes we choose taking into account specific properties of the Nariai–Bertotti–Robinson spacetimes.

In general relativity, the problem of wormholes is not more exotic than that of black holes. In this talk we consider a simple toy model which is still far from perfection which could however be useful in better comprehension of the magnitude of the wormhole problem.

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The Nariai–Bertotti–Robinson (NBR) solution[2, 7, 8, 9, 10] (about the result of Robinson[9] see however Ref. 3) can be described as $ds^2 = e^{2\alpha(r)}dt^2 - e^{-2\alpha(r)}dr^2 - \lambda^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2)$ where $e^{2\alpha} = (k^2 - \Lambda)r^2 + Br + C$ and $\lambda = \frac{1}{\sqrt{\Lambda+k^2}}$, B and C being arbitrary constants, Λ the cosmological constant, and k , the (constant) electromagnetic field intensity. The electromagnetic sources in Einstein's equations correspond to the four-potential $\mathcal{A} = \sqrt{\frac{8\pi}{\kappa}} \left(akr dt + \frac{kb}{(k^2+\Lambda)} \cos\vartheta d\varphi \right)$: $T_{em} = \frac{k^2}{\kappa} (\theta^{(0)} \otimes \theta^{(0)} - \theta^{(1)} \otimes \theta^{(1)} + \theta^{(2)} \otimes \theta^{(2)} + \theta^{(3)} \otimes \theta^{(3)})$ with $a = \sin\psi$, $b = \cos\psi$, ψ being an arbitrary constant, while (see a general discussion in Ref. 4) $E = *(\theta^{(0)} \wedge *F) = \mathcal{A}_{0,r}\theta^{(1)}$, $B = *(\theta^{(0)} \wedge F) = \frac{\Lambda+k^2}{\sin\vartheta} \mathcal{A}_{3,\vartheta}\theta^{(1)}$ where $\theta^{(0)} = e^\alpha dt$, $\theta^{(1)} = e^{-\alpha} dr$, $\theta^{(2)} = \lambda d\vartheta$, $\theta^{(3)} = \lambda \sin\vartheta d\varphi$.

We consider pieces of NBR solutions with two horizons (null compact hypersurfaces along whose generatrices $ds^2 = 0$, while $|\frac{dt}{dr}| \rightarrow \infty$ on the horizons). When $k^2 > \Lambda > -k^2$, the two horizons are at $r = \pm r_0 = \pm 1/\sqrt{k^2 - \Lambda}$ with a non-stationary band between them and static regions outside. Alternatively, when $k^2 < \Lambda$, the horizons are at $r = \pm r_0 = \pm 1/\sqrt{\Lambda - k^2}$ and spacetime is static between them and non-stationary outside. We now write these solutions in synchronous coordinates (see the definition in the footnote on p. 62 of Ref. 4):

$$ds^2 = dT^2 - [\mathcal{E}^2 - g_{00}(r(T, R))]dR^2 - \frac{1}{\Lambda + k^2}[d\vartheta^2 + \sin^2\vartheta d\varphi^2]. \quad (1)$$

Here \mathcal{E} is energy per unit mass of the geodesically moving test particle identified with the observer. On horizons where $g_{00} = 0$, no singularities and degeneracy appear in the metric coefficients. (This makes it unnecessary to apply the intrinsic prescription in the Barrabès and Israel formalism. At the horizon there is then used a thin null shell.[1, 5, 6]) This description also gives a unique junction of spacetimes and enables the standard causal treatment of an infinite sequence of universes in the Penrose diagram. In the case $k^2 > \Lambda > -k^2$, $g_{00}(r) = (k^2 - \Lambda)r^2 - 1$; in the case $k^2 < \Lambda$, $g_{00}(r) = 1 - (\Lambda - k^2)r^2$.

As the outside worlds we consider the Reissner–Nordström–Kottler (RNK) solutions[10] (those of Reissner–Nordström, but with the cosmological term),

$$ds^2 = dT^2 - [\mathcal{E}^2 - g_{00}(r)]dR^2 - r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2) \quad (2)$$

with $g_{00} = 1 - \frac{2m_1}{r} + \frac{e_1^2}{r^2} - \frac{1}{3}\Lambda_1 r^2$ and $g_{00} = 1 - \frac{2m_2}{r} + \frac{e_2^2}{r^2} - \frac{1}{3}\Lambda_2 r^2$, $r = r(T, R)$. They are to be joined *via* wormholes which belong to the NBR spacetimes,

(1), with $g_{00}(r) = \left(\frac{k^2 - \Lambda}{k^2 + \Lambda}\right) [(\Lambda + k^2)r^2 - 1]$ (there is also $r(T, R)$, but with another dependence than in RNK), when the cases $\Lambda > k^2$ and $k^2 > \Lambda > -k^2$ are unified via a scales change in r , so that the horizons correspond to $r = \pm\lambda = \pm\frac{1}{\sqrt{\Lambda + k^2}}$ (the minus sign does not spoil our considerations since it can be inverted when we consider the junction of the NBR-wormhole with the ‘second’ RNK world at this horizon). At the horizons in synchronous coordinates we put in (2) $g_{00} = 0$ and substitute instead of r , $\bar{r} = \bar{r}(m_1, e_1, \Lambda_1) = \bar{r}(m_2, e_2, \Lambda_2)$, corresponding to anyone of the (three) horizons of RNK, while in NBR (1) the only change at the horizon is to put $g_{00} = 0$. Hence we conclude that

$$[\tilde{g}_{\alpha\beta}] = 0 \quad \Rightarrow \quad \frac{1}{\sqrt{k^2 + \Lambda}} = \bar{r}. \quad (3)$$

The electromagnetic stress-energy tensors read $T_{\text{NBR}} = \frac{k^2}{\kappa} \{dT \otimes dT - \mathcal{E}^2 dR \otimes dR + \frac{1}{k^2 + \Lambda} (d\vartheta \otimes d\vartheta + \sin^2 \vartheta d\varphi \otimes d\varphi)\}$ and $T_{\text{RNK}} = \frac{e_{1,2}^2}{\kappa \bar{r}^4} \{dT \otimes dT - \mathcal{E}^2 dR \otimes dR + \bar{r}^2 (d\vartheta \otimes d\vartheta + \sin^2 \vartheta d\varphi \otimes d\varphi)\}$, thus

$$[T_{\mu\nu}] = 0 \quad \Rightarrow \quad k^2 = \frac{e_{1,2}^2}{\bar{r}^4}. \quad (4)$$

Taking into account (3) and (4), we see that $\Lambda = \frac{\bar{r}^2 - e_{1,2}^2}{\bar{r}^4}$ and $e_{1,2}^2 = e^2 = \frac{k^2}{(k^2 + \Lambda)^4}$, thus the charges observed from opposite entrances of the wormhole coincide up to the sign.

The interior of such wormholes is a non-stationary region if $k^2 > \Lambda > -k^2$ or a static region if $\Lambda > k^2$. These types of wormholes are observed in one universe as black holes, in another universe (or on another spacetime sheet of the former universe) as white holes, though there is no singularity which should correspond to usual black holes, since they belong here to NBR. Observers in these two adjacent RNK universes would conclude that the wormhole has an electric charge with the same absolute value, but opposite signs in different universes (or the similar situation with the magnetic charge); they would also measure a non-zero positive mass of the wormhole, but this mass in general will be different for observers in different universes (together with the different values of the cosmological constant corresponding to the respective worlds).

It is comparatively easy to construct examples of Penrose diagrams’ hybridization resulting in a connection of two RNK worlds *via* a NBR-wormhole.

They show that the wormholes under consideration are traversable only in one direction (one-way wormholes) taking the traveller to another sheet of spacetime (behind the future infinity of the abandoned world); this is also visualized by diagrams using synchronous coordinates. Of course, the Penrose diagrams' hybridization cannot be simply shown on one piece of paper since the RNK singularities and adjacent sectors require more space than there is at one's disposal on one sheet so that one has to identify some boundaries of these sectors without mixing them with those pertaining to the NBR-wormhole. Therefore we do not show such hybridized diagrams here.

Naturally, the junction of RNK worlds *via* static part of NBR-wormhole can be also done not on horizons, but in the outside parts of RNK worlds and of NBR spacetime, thus permitting to consider construction of two-way wormholes; in this case it is natural to glue together only static regions of both spacetimes. This requires the use of more complicated prescriptions for junction, and we do not come in these details leaving them to another publication. The NBR solution is chosen in this talk as a convenient tool to construct wormholes since it already has the necessary properties for modelling them due to the angular part of the NBR metric.

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